**Lecture 9.**

**Differentials. Properties of differential. Derivatives of Higher Orders. Leibniz formula. Higher-order differentials.**

 **Definition.** The **differential** (first order) of a function  is the principal part of its increment, which part is linear relative to the increment  of the independent variable $x$. The differential of a function is equal to the product of its derivative by the differential of the independent variable

 $dy=y'dx$, (1)

hence

$ y^{'}=\frac{dy}{dx}$. (2)

 **Example.** Find the increment and differential of a function $y=3x^{2}-x.$

 **Solution:**

*First method*:

$$∆y=3(x+∆x)^{2}-\left(x+∆x\right)-3x^{2}+x$$

or

 $∆y=\left(6x-1\right)∆x+3(∆x)^{2}$.

Hence,

$$dy=\left(6x-1\right)∆x=\left(6x-1\right)dx.$$

 *Second method:*

$$y^{'}=6x-1;$$

By (1) we get

$$dy=y^{'}dx=\left(6x-1\right)dx.$$

**Definition of higher derivatives.** A derivative of the second order, or *second derivative*, of the function $y=f(x)$ is the derivative of its derivative; that is, $y^{''}=\left(y^{'}\right)^{'}.$ The second derivative may be denoted as

$$y^{''}, or \frac{d^{2}y}{dx^{2}}, or f ''\left(x\right).$$

If $x=f(t)$ is the law of rectilinear motion of a point, then $\frac{d^{2}x}{dt^{2}} $ is the acceleration of this motion.

Generally, the *n-*th *derivative* of a function $y=f(x)$ is the derivative of a derivative of order (*n-1).* For the n-th derivative we use the notation

**Leibniz rule.** If the functions $u=f\left(x\right) and ϑ=g(x)$ have derivatives up to the *n-*th order inclusive, then to evaluate the *n-*th derivative of a product of these functions we can use Leibniz rule (or formula):



Since,

 

then we have



consequently, we can write Leibniz formula in the form



**Higher order differentials:** A second order differentialis the differential of a first order differential:

$d^{2}=d\left(dy\right).$ (10)

We similarly define the differentials of the third and higher orders.

If $y=f(x)$ and $x$ is an independent variable, then

